Directions: You have **90 minutes** to answer the following questions. **You must show all your work** as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator. The last page contains formulas that you might find useful. You may tear that page out. You may choose to have only 8 problems (each worth 12 points) graded or 9 problems (each worth 11) graded or 10 problems (each worth 10) graded. For example if you do **not** want to have Problem 6 graded, you **MUST** put an “X” in the Points section of Problem 6.

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(1) (Homework) The quantity (in hundreds of yards per month) of a certain silk that is sold by a retailer at a price of $p$ dollars per yard is $Q = f(p)$.

(a) Which of the following sentences is a correct interpretation in terms of price and quantity of the statement $f(6.2) = 2.5$?

(i) If they set the price of silk to $6.20 per yard, then they sell 2.5 yards per month.

(ii) The retailer sells 250 yards a month of the silk, if its selling price is $6.20 a yard.

(iii) If the price of silk increases by a dollar, then they can sell 250 yards more of the fabric per year.

(iv) If the price of silk increases by a dollar, then they can sell 2.5 yards more of the fabric per day.

(v) If the price of the silk is $2.50 per yard, then the retailer sells 620 yards of this fabric a month.

(b) Which of the following sentences is a correct interpretation in terms of price and quantity of the statement $f'(6.2) = -1$?

(i) If the retailer lowers the price of the silk by a dollar per yard, then they can sell approximately 6.2 yards more a month.

(ii) If the price of the silk increases to $7.2 per yard, then the retailer can sell approximately 350 yards a month.

(iii) If the price of the silk increases to $7.2 per yard, then the retailer can sell approximately 1.5 yards a day.

(iv) If the retailer lowers the price of the silk by a dollar per yard, then they can sell approximately 350 yards a month.

(v) If the retailer lowers the price of the silk by a dollar per yard, then they can sell approximately 100 yards less a month.

(c) What are the units of $f'(p) = \frac{dQ}{dp}$?
(2) Determine whether each of the following statements is True or False. (You do not have to explain.)

(a) Average velocity is \( \dfrac{\text{total distance traveled}}{\text{time elapsed}} \).

(b) Let \( h \) be any function such that \( h(5) = 7 \), then \( \lim_{x \to 5} \left[ 2h(x) - 1 \right] = 13 \).

(c) If \( \lim_{x \to 0} f(x) = \infty \) and \( \lim_{x \to 0} g(x) = \infty \), then \( \lim_{x \to 0} \left[ f(x) - g(x) \right] = 0 \).

(d) If \( f \) is not defined at 1, then the line \( x = 1 \) must be a vertical asymptote of \( y = f(x) \).

(e) If \( f \) is differentiable at 5, and \( f(5) = 2 \), then \( \lim_{x \to 5} \left[ 3f(x) + 1 \right] = 7 \).
Each of the graphs below shows the position of a particle moving along the x-axis as a function of time, t, where 0 \leq t \leq 5. The vertical scales of the graphs are the same.

During this time interval, which particle(s)

(a) has the least initial velocity?

(b) has the greatest initial velocity?

(c) has the greatest average velocity?

(d) has positive acceleration for the entire period?

(e) has zero acceleration for some period?
(4) (Page 84, Problems 23–32) For each of the functions (a)–(d), find a function from (I)–(VIII) which could be its derivative.

**Functions**

(a) \[ y = x^3 \]

(b) \[ y = x^2 + 1 \]

(c) \[ y = \frac{1}{x} \]

(d) \[ y = \cos(x) \]

**Derivatives**

(I) \[ y' = 3x^2 \]

(II) \[ y' = 2x \]

(III) \[ y' = -\frac{1}{x^2} \]

(IV) \[ y' = -\sin(x) \]

(V) \[ y' = \sin(x) \]

(VI) \[ y' = 0 \]

(VII) \[ y' = 0 \]

(VIII) \[ y' = 1 \]

- The derivative of function (a) is:
- The derivative of function (b) is:
- The derivative of function (c) is:
- The derivative of function (d) is:
(5) (Page 69, Problems 30–36) Evaluate each of the following limits. Here $a$ and $b$ are positive constants. You must show your work.

(a) \[ \lim_{x \to \infty} \frac{ae^{-x} + \frac{1}{b}}{be^{-x} + \frac{2}{b}} \]

(b) \[ \lim_{x \to 0} \frac{(a + \frac{x}{b})^2 - a^2}{x} \]
(6) (Page 62, Problem 18) Consider the function \( y = f(t) \) shown in the figure, which represents the displacement of an object along the \( y \)-axis. For each of the following pairs of numbers, decide which one is larger. (Circle the larger one. You do not need to justify it.)

(a) the instantaneous velocity at \( t = 3 \) \hspace{1cm} OR \hspace{1cm} the acceleration at \( t = 3 \)?

(b) the average velocity from \( t = 1 \) to \( t = 5 \) \hspace{1cm} OR \hspace{1cm} the average velocity from \( t = 1 \) to \( t = 6 \)?

(c) the instantaneous velocity at \( t = 2 \) \hspace{1cm} OR \hspace{1cm} the average velocity from \( t = 2 \) to \( t = 3 \)?

(d) \( \frac{f(2)}{2} \) \hspace{1cm} OR \hspace{1cm} average velocity from \( t = 1 \) to \( t = 2 \)?
(7) (Worksheet 1) Using the graph of $f$ and $g$, evaluate each expression, or state if it is undefined.

(a) $\lim_{t \to 3} [f(t) + g(t)] =$

(b) $\lim_{t \to 3^+} \frac{f(t)}{g(t)} =$

(c) $\lim_{t \to 3^-} [f(t)]^3 =$

(d) $\lim_{t \to 2} [f'(t) - g(t)] =$

(e) $\lim_{t \to \frac{1}{3}} f(g(t)) =$
(8) (Worksheet 1) For what value(s) of \(a\) is the function

\[
f(x) = \begin{cases} 
-6 & \text{if } x < a \\
\ln(x)\left(1 - \ln(x)\right) & \text{if } x \geq a 
\end{cases}
\]

continuous at \(x = a\)? Show all your work.
A function $f$ satisfies the following conditions:

$f(5) = 20$, \quad $f'(5) = 2$, \quad and \quad $f''(x) < 0$ \quad for \quad $-1 \leq x \leq 8$.

(a) Which of the following are possible values for $f(1)$?

(i) 11

(ii) 13

(iii) 12

(b) Use the tangent line approximation to estimate the value of $f(5.3)$. Show your work.
(10) (Page 94, Problem 20) A continuous function defined for all $x$ has the following properties:

- $f$ is increasing
- $f''(x) < 0$ for all $x$
- $f(5) = 2$
- $f'(5) = \frac{1}{2}$.

(a) Sketch a graph of $f$.

(b) How many zeroes does $f$ have?

(c) What is $\lim_{x \to -\infty} f(x)$?

(d) Is it possible that $f'(1) = 1$?
Formulas you might find useful

- The derivative of a function
  \[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]

- \( \frac{d}{dx} x^n = nx^{n-1} \)

- \( \frac{d}{dx} e^x = e^x \)