Directions: You have 90 minutes to answer the following questions. You must show all your work as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator. The last page contains formulas that you might find useful. You may tear that page out. You may choose to have only 8 problems (each worth 12 points) graded or 9 problems (each worth 11) graded or 10 problems (each worth 10) graded. For example if you do not want to have Problem 6 graded, you MUST put an “X” in the Points section of Problem 6.

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(1) Determine whether each of the following statements is True or False. (You do not have to explain.)

(a) If a differentiable function $f$ is periodic, then $f'$ is also periodic.

(b) If $f(x)$ and $g(x)$ are both decreasing, then $f(g(x))$ is also decreasing.

(c) If $f$ is a negative decreasing function, then $\frac{1}{f}$ is also decreasing.

(d) If $\lim_{x \to 0} f(x) = \infty$ and $\lim_{x \to 0} g(x) = \infty$, then $\lim_{x \to 0} [f(x) - g(x)] = 0$.

(e) If the local linearization of $f(x)$ at $x = 1$ is given by $L_f(x) = -3(x - 1)$, and the local linearization of $g(x)$ at $x = 1$ is $L_g(x) = 5(x - 1)$, then 
\[ \lim_{x \to 1} \frac{f(x)}{g(x)} = -0.6. \]
(2) (Page 173, Problem 4) The graph of $g'$ (not $g$) is given in the following figure. It is also known that $g(0) = 0$.

![Graph of $g'(t)$](image)

Fill in the blanks. (You do not have to explain.)

(a) $g$ has an inflection point(s) at $t =$__________.

(b) $g$ is concave up on the interval(s) ________.

(c) $g$ has a local minimum at $t =$__________.

(d) Is $g$ positive or negative on the interval $(-2, 0)$? ________.

(e) $\lim_{t \to 0} \frac{g(t)}{t^2} =$__________.
(3) (Page 179, Problem 20) Let
\[ f(x) = x^4 + ax^2 + b. \]

(a) Under what conditions on the constants \( a \) and \( b \) does this function have exactly one critical point?

(i) What is the critical point?

(ii) Is it a local maximum, a local minimum, or neither?

(b) Under what conditions on \( a \) and \( b \) does this function have three critical points?
(4) (Page 187, Problem 26) Let

\[ f(x) = \frac{\ln x}{\sqrt{x}}. \]

(a) (7 points) Find the x-coordinate where the function assumes its global maximum on the interval \([e, \infty)\). What is the global maximum value of the function?

(b) (3 points) Explain why this function does not have a global minimum on the interval \([e, \infty)\).
(5) (Page 187, Problems 21–26) Find the absolute maximum and absolute minimum of \( z(t) = t - 2 \cos(t) \) on the closed interval \([−\pi, \pi] \).
(6) (Page 150, Problem 64) Which of the following functions are tangent to the line $L(x) = b(x - 1)$ at $x = 1$? Here $b$ is a non-zero constant. Circle all correct choices.

(a) $y = \frac{2bx}{x^2 + 1}$

(b) $y = \arctan(bx - b)$

(c) $y = be^{(x-1)}$
(7) (Page 201, Problem 8) You need to manufacture a cylindrical pot, without a top, with a volume of 1 ft$^3$. The cylindrical part of the pot is to be made of aluminum, the bottom of copper. Copper is five times as expensive as aluminum. What dimensions would minimize the total cost of the pot?
(8) (MA 1012, Page 173, Problem 5) The function $f$ is given by its graph (see the picture below). The function $g$ is given by the formula $g(x) = x^2 - 2x - 1$.

Find the critical points of the composite function $h(x) = f(g(x))$. 
(9) (Page 158, Problems 15–17) Calculate the following limit. Show your work.

\[ \lim_{x \to 1} \frac{1 - x + \ln x}{1 + \cos(\pi x)} \]
(10) (Page 153, Problem 6) Use the tangent line approximation to the graph of

\[ h(t) = e^{-3t^2 + 3} \]

at \( t = 1 \) to estimate the value of \( h(0.9) \). Show all your work.
Formulas you might find useful

- The derivative of a function
  \[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

- Some rules of differentiation
  \[ \frac{d}{dx} (f(x)g(x)) = f'(x)g(x) + f(x)g'(x) \]
  \[ \frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \]
  \[ \frac{d}{dx} f(g(x)) = f'(g(x))g'(x) \]

- The tangent line approximation for a function \( f \) for \( x \) near \( a \) is given by
  \[ f(x) \approx f(a) + f'(a)(x-a) \]

- **Geometry Formulas**
  Here \( A \) is the area, \( C \) is the circumference, \( V \) is the volume, \( S \) is the surface area, \( h \) is the height and \( r \) is the radius.
  Circle: \( A = \pi r^2; \ C = 2\pi r \)
  Cylinder: \( V = \pi r^2h, S = 2\pi rh \)
  Cone: \( V = \frac{1}{3}\pi r^2h \)
  Sphere: \( V = \frac{4}{3}\pi r^3, S = 4\pi r^2 \)

- **Differentiation formulas**

| \( \frac{d}{dx} (x^n) = nx^{n-1} \) | \( \frac{d}{dx} (e^x) = e^x \) | \( \frac{d}{dx} (a^x) = (\ln a)a^x \) |
| \( \frac{d}{dx} (\ln x) = \frac{1}{x} \) | \( \frac{d}{dx} (\sin(x)) = \cos x \) | \( \frac{d}{dx} (\cos(x)) = -\sin x \) |
| \( \frac{d}{dx} (\arcsin(x)) = \frac{1}{\sqrt{1-x^2}} \) | \( \frac{d}{dx} (\sec(x)) = \sec x \tan x \) | \( \frac{d}{dx} (\csc(x)) = -\csc x \cot x \) |
| \( \frac{d}{dx} (\sinh(x)) = \cosh(x) \) | \( \frac{d}{dx} (\arccos(x)) = -\frac{1}{\sqrt{1-x^2}} \) | \( \frac{d}{dx} (\csc(x)) = -\csc x \cot x \) |
| \( \frac{d}{dx} (\cosh(x)) = \sinh(x) \) | \( \frac{d}{dx} (\arctan(x)) = \frac{1}{1+x^2} \) | \( \frac{d}{dx} (\tanh(x)) = \frac{1}{\cosh^2(x)} \) |