Directions: You have **90 minutes** to answer the following questions. **You must show all your work** as neatly and clearly as possible and indicate the final answer clearly. You may use a calculator. The last page contains formulas that you might find useful. You may tear that page out. You may choose to have only 8 problems (each worth 12 points) graded or 9 problems (each worth 11) graded or 10 problems (each worth 10) graded. For example if you do **not** want to have Problem 6 graded, you **MUST** put an “X” in the Points section of Problem 6.

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(1) (MA1012) Determine whether each of the following statements is True or False. (You do not have to explain.)

(a) If an everywhere differentiable function $f$ is even, then $f'(0) = 0$.

(b) If $f$ is a differentiable function with $f(0) = 0$ and $f'(0) = 3$, then
\[ \lim_{x \to 0} [2f(x) - 1] = 5. \]

(c) If $f$ is a negative decreasing function, then $f(f(x))$ is also decreasing.

(d) If $\lim_{x \to \infty} f(x) = \infty$ and $\lim_{x \to \infty} g(x) = \infty$, then $\lim_{x \to \infty} [f(x) - g(x)] = 0$.

(e) $\frac{d}{dx} |x^2 - 2| = |2x|$. 
(2) (Page 173, Problem 4) The graph of $f'$ (not $f$) is given in the following figure. It is also known that $f(1) = 0$.

![Graph of $f'$](image.jpg)

Fill in the blanks. (You do not have to explain.)

(a) $f$ has local maximum at $t =$__________.

(b) $f$ is concave up on the interval(s) __________.

(c) $f'$ has local minimum at $t =$__________.

(d) Is $f$ positive or negative on the interval $(3, 4)$? __________.

(e) $f$ has inflection point(s) at $t =$__________.
(3) Determine whether each of the following statements is True or False. (You do not have to explain.)

(a) If a function $f$ has a critical point at $p$, then $f'(p)$ must be 0.

(b) There exists a twice differentiable function $g$ such that $g(x) > 0$, $g'(x) < 0$ and $g''(x) > 0$ for all $x$.

(c) If $h''(2) = 0$, then $(2, h(2))$ is an inflection point of the curve $y = h(x)$.

(d) There exists a differentiable function $f$ such that $f(1) = 5$, $f(3) = 2$, $f'(x) < 0$ and $f''(x) < 0$ for all $x$.

(e) If a continuous function $f$ has a global maximum at $p$ on the closed interval $[-1, 3]$, then $f'(p)$ must be 0.
(4) (Page 186, Problem 7) The number of salmon swimming upstream to spawn is approximated by

\[ S(T) = -T^3 + 3T^2 + 360T + 5000, \quad 6 \leq T \leq 20, \]

where \( T \) represents the temperature of the water in degrees Celsius. Use calculus to find the water temperature that produces the maximum number of salmon swimming upstream. You must show all your work.
(5) (Page 153, Problem 9) Let $a$ and $b$ be positive constants. Find the local linearization of the function

$$f(x) = \frac{1}{\sqrt{3a + bx}}$$

at $x = \frac{a}{b}$. Show your work and simplify your answer.
(6) (Page 158, Problem 11) Use l’Hopital’s rule to determine which function dominates as $x \to \infty$.

\[
\ln(x + 3) \quad \text{OR} \quad x^{0.2}
\]
(7) (Page 201, Problem 6) Find two nonnegative numbers $x$ and $y$ for which $2x + y = 30$, such that $xy^2$ is maximized.
(8) (Page 179, Problem 11) Find the value of each of the parameters $a$ and $b$ so that the function

$$g(x) = \frac{ax^2}{e^{bx}}$$

has a local maximum value of $e^2$ at $x = 1$. Show all your work.
(9) (Page 158, Problems 15–17) Let $a$ be a positive real number. Find

$$
\lim_{x \to 0} \frac{e^x - x - 1}{x \sin(ax)}.
$$

Show your work.
(10) (Page 202, Problem 22) A hunter is at a point on a river bank. He wants to get to his cabin, located 3 miles north and 8 miles west. (See the figure.) He can travel 5 mph along the river but only 2 mph on this very rocky land. How far should he go along the riverbank in order to reach the cabin in minimum time?
Formulas you might find useful

- The derivative of a function
  \[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]

- Some rules of differentiation
  \[
  \frac{d}{dx} (f(x)g(x)) = f'(x)g(x) + f(x)g'(x) \\
  \frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \\
  \frac{d}{dx} f(g(x)) = f'(g(x))g'(x)
  \]

- The tangent line approximation for a function \( f \) for \( x \) near \( a \) is given by
  \[ f(x) \approx f(a) + f'(a)(x - a) \]

- Geometry Formulas
  Here \( A \) is the area, \( C \) is the circumference, \( V \) is the volume, \( S \) is the surface area, \( h \) is the height and \( r \) is the radius.
  Circle: \( A = \pi r^2; \ C = 2\pi r \)
  Cylinder: \( V = \pi r^2h, \ S = 2\pi rh \)
  Cone: \( V = \frac{1}{3}\pi r^2h \)
  Sphere: \( V = \frac{4}{3}\pi r^3, \ S = 4\pi r^2 \)

- Differentiation formulas

| \( \frac{d}{dx} (x^n) = nx^{n-1} \) | \( \frac{d}{dx} (e^x) = e^x \) | \( \frac{d}{dx} (a^x) = (\ln a)a^x \) |
| \( \frac{d}{dx} (\ln x) = \frac{1}{x} \) | \( \frac{d}{dx} (\sin(x)) = \cos x \) | \( \frac{d}{dx} (\cos(x)) = -\sin x \) |
| \( \frac{d}{dx} (\arcsin(x)) = \frac{1}{\sqrt{1 - x^2}} \) | \( \frac{d}{dx} (\sec(x)) = \sec x \tan x \) | \( \frac{d}{dx} (\cot(x)) = -\csc^2 x \) |
| \( \frac{d}{dx} (\sinh(x)) = \cosh(x) \) | \( \frac{d}{dx} (\cosh(x)) = \sinh x \) | \( \frac{d}{dx} (\csc(x)) = -\csc x \cot x \) |
| \( \frac{d}{dx} (\arccos(x)) = -\frac{1}{\sqrt{1 - x^2}} \) | \( \frac{d}{dx} (\arctan(x)) = \frac{1}{1 + x^2} \) | \( \frac{d}{dx} (\tanh(x)) = \frac{1}{\cosh^2(x)} \) |